

Comment on “Reaction Imaging with Interferometry”

In a recent Letter [1], Feagin and Han analyzed scattering into a double-slit interferometer with target-fragment recoil detection as a monitor of quantum correlation and entanglement in few-body reaction amplitudes. In particular, they claim to extend Wootters and Zurek’s analysis [2] (on quantum interference and which-way information in a double-slit experiment) to a scattering problem. Here we show that the analysis in [1] is incorrect.

Reference [1] considered the quantum entanglement of the relative motion of various recoiling fragments with the scattered projectile. However, in doing so the authors neglect other aspects of quantum entanglement. In particular, the starting point of Ref. [1], Eq. (1) for the wave function of the scattered projectile after entering the interferometer, is incorrect. This is because it does not take account of the fact that different outgoing states of the scattered projectile are entangled with different momentum states of the center of mass motion of particles 2 and 3. Taking this into account gives the correct wave function as [analogous to their Eq. (1)]

$$\chi_n = P_n^{-1/2} [f_{n+} \exp(-i\mathbf{r}_1 \cdot \mathbf{k}_{f+}) \otimes |\mathbf{K}_c - \mathbf{k}_{f+}\rangle_{23} + f_{n-} \exp(-i\mathbf{r}_1 \cdot \mathbf{k}_{f-}) \otimes |\mathbf{K}_c - \mathbf{k}_{f-}\rangle_{23}], \quad (1)$$

where $|\cdot\rangle_{23}$ denotes the wave function of the center of mass motion of particles 2 and 3, $\mathbf{r}_1$ is the position of particle 1, $\mathbf{K}_c$ is the conserved total momentum of the scattering system, and all other notation is as in Ref. [1]. As is clear from Eq. (1), by measuring $|\cdot\rangle_{23}$, one can readily tell through which slit the scattering projectile has gone. As a result of this complete which-way information, there are no quantum interference effects in the double-slit experiment described in Ref. [1] if, in accord with Ref. [1], both the projectile and the target atom are initially in momentum eigenstates. Therefore, contrary to the claims in Ref. [1], their two-slit imaging cannot determine the relative phase between different scattering amplitudes for various momentum components. That is, Eq. (2) in Ref. [1] is incorrect as well.

The authors of Ref. [1] further claim that the interference patterns can be maintained if at least one of the target fragments is undetected. This is not the case. As is well known, even if we make no measurements on any recoil fragments (so that the potentially available which-way information is not extracted at all), there are still no inter-

ference patterns [3–5]. That is, neglecting the which-way information from $|\cdot\rangle_{23}$ is equivalent to tracing over $|\cdot\rangle_{23}$, so that the scattered projectile should, in this case, be described by a density matrix

$$\rho_n(\mathbf{r}_1, \mathbf{r}'_1) = P_n^{-1} \{ |f_{n+}|^2 \exp[-i(\mathbf{r}_1 - \mathbf{r}'_1) \cdot \mathbf{k}_{f+}] + |f_{n-}|^2 \exp[-i(\mathbf{r}_1 - \mathbf{r}'_1) \cdot \mathbf{k}_{f-}] \}. \quad (2)$$

The authors of Ref. [1] also considered a limiting case in which the mass of the recoil ion m_3 is infinitely heavy. In this case interference in the observed double-slit pattern is possible since one cannot define a definite $\mathbf{K}_c$. Indeed, as long as particle 3 is macroscopic, it can introduce sufficient momentum uncertainty to disallow the which-way information and allow for the interference pattern. Alternatively, momentum uncertainty can be introduced by replacing the plane wave state of the target atom by a momentum wave packet. However, for the parameters in Ref. [1], this requires a pulsed target atom beam whose duration is far shorter than current capabilities, i.e., on the order of a picosecond.

In conclusion, the analysis in [1] is incorrect, except for a limiting case of an infinitely heavy recoil ion. In the atomic case, then, anchoring the recoil ion to a surface would provide a realistic model for observing the interferences that the authors seek.

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