Controlled quantum-state transfer in a spin chain

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Control of the transfer of quantum information encoded in quantum wave packets moving along a spin chain is demonstrated. Specifically, based on a relationship with control in a paradigm of quantum chaos, it is shown that wave packets with slow dispersion can automatically emerge from a class of initial superposition states involving only a few spins, and that arbitrary unspecified traveling wave packets can be nondestructively stopped and later relaunched with perfection. The results establish an interesting application of quantum chaos studies in quantum information science.

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I. INTRODUCTION

Great effort is being devoted to studies of spin chains as promising "quantum wires" for quantum information transfer. With spin chains, a quantum state can be transferred without requiring an interface between the communication channel and a quantum computer [1], i.e., quantum information can be transferred and processed with the same hardware. Spin chains also allow for quantum computing with an always-on interaction [2,3], even in the presence of a global control field [3]. The latest experimental progress on fabrication and characterization of atomic spin chains was reported in Ref. [4]. Spin chain Hamiltonians may be also realized by atomic gas in an optical lattice.

Perfect state transfer in spin chains might occur under special circumstances [5–7]. However, in the general case, dispersion effects often degrade the transmission fidelity and improving the fidelity becomes a central issue. Notably, it has been proven that the transmission fidelity can be significantly improved if the receiver stores the received signal in a large quantum memory before decoding [8]. Another general approach to high-fidelity quantum-state transfer advocates the use of quantum wave packets to encode the quantum state of a qubit [9,10]. This approach is important because dispersion of wave packets can be insignificant. In particular, Osborne and Linden [9] have shown that high transmission fidelity can be achieved by exploiting, if attainable, a Gaussian wave packet whose shape is well preserved. The slow dispersion of a wave packet can be further suppressed by applying a static parabolic (hence global) magnetic field [11].

In the context of the wave packet approach to quantum-state transfer, we focus below on two questions: (1) How can one create spin wave packets with certain desired features? (2) How can one control the motion of a quantum wave packet in a spin chain so that the packet can be stopped at an arbitrary time, held, and then restarted later, without loss of quantum information? Such type of controlled quantum-state transfer, if possible, should be a highly valuable tool in a variety of situations, e.g., cases in which the information receivers need additional waiting time to repair a quantum memory, or to prepare for a time window of high transmission fidelity. The importance of stoppable quantum-state transfer may be also appreciated by noting the analogy to the potential impact of the stopping of light [12] in quantum information science. Further, a working scenario for the stopping and perfect relaunching of quantum-state evolution of a spin chain should be also of considerable interest in the context of perfect quantum-state reconstruction and perfect quantum-state storage in systems of interacting qubits [13].

In this paper we first show that by optimizing a particular transport property using quantum superposition states comprising only a few spins (e.g., four or five), wave packet pairs with some highly desired features emerge automatically from the ensuing dynamics. We then demonstrate that by applying a sequence of pulsed parabolic magnetic fields one can manipulate these wave packets, stopping them and later relaunching the traveling wave packets without individually addressing the spins. As shown below, the stopping, followed by relaunching, can in principle perfectly preserve the quantum information being transferred. This is made possible by taking advantage of powerful relationships between controlling spin dynamics and controlling quantum diffusion dynamics in a paradigm of quantum chaos.

This paper is organized as follows. In Sec. II we introduce a mapping between a Heisenberg spin chain kicked by a parabolic magnetic field and a paradigm in quantum chaos [14,15]. In Sec. III we propose a conceptually simple approach to the creation of spin wave packet pairs moving along the spin chain with slow dispersion and other desired features. The key result of this work is in Sec. IV, where stopping and relaunching spin wave packets are studied both numerically and analytically. Section V concludes this paper.

II. HEISENBERG SPIN CHAIN IN A PULSED MAGNETIC FIELD AND THE DELTA-KICKED ROTOR

Consider then an open-ended Heisenberg chain of N spins in a constant magnetic field B and subject to a parabolic δ-pulsed magnetic field. The Hamiltonian is given by

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$$H = -\frac{J}{2}\sum_{n=1}^{N-1} \sigma_n \cdot \sigma_{n+1} - B\sum_{n=1}^{N} \sigma_n^z + \sum_j \delta(t - jT_0)\sum_{n=1}^{N} \sigma_n^z C_j \frac{(n-n_0)^2}{2}, \quad (1)$$

where $\sigma \equiv (\sigma^x, \sigma^y, \sigma^z)$ are the Pauli matrices, J is the nearest-neighbor spin-spin interaction constant, C_j and n_0 are the coefficient and minimum location of the parabolic kicking field, and T_0 is the kicking period. Below, we denote the $n=1$ ($n=N$) spin as the left (right) end of the chain. The constant field B lifts the system degeneracy and the dynamics is restricted to a subspace with fixed total polarization S_z defined as

$$S_z \equiv \sum_{n=1}^{N} \sigma_n^z. \quad (2)$$

Throughout this work we consider only the subspace of $S_z = 2-N$.

Let $|\mathbf{m}\rangle$ be one of the basis states, with the mth spin up and all other spins down. The propagator for the time period $[jT_0 - 0^+, (j+1)T_0 - 0^+]$ is $\hat{V}(2JT_0)\hat{U}(C_j)$. Here $\hat{U}(C_j)$ represents the action due to the delta pulse, with

$$\langle \mathbf{m}|\hat{U}(C_j)|\mathbf{n}\rangle = \exp[-i(C_j/2)(n-n_0)^2]\delta_{mn}. \quad (3)$$

The term $\hat{V}$ stems from the evolution inherent in the Heisenberg interaction. An important recent study [14] has shown that, in the $N \to +\infty$ limit (and apart from some irrelevant phase),

$$\langle \mathbf{m}|\hat{V}(2JT_0)|\mathbf{n}\rangle \approx i^{(m-n)} J_{(m-n)}(2JT_0), \quad (4)$$

where $J_{(m-n)}$ is an ordinary Bessel function. The analytical behavior of $\hat{U}(C_j)$ and $\hat{V}(2JT_0)$ is therefore completely in parallel with that associated with the propagator of the δ-kicked rotor (DKR) (the best known model in quantum chaos [16]) with Hamiltonian

$$H_{\mathrm{DKR}} = (\hat{P} - P_0)^2/2 - K\cos(\theta)\sum_j \delta(t-j). \quad (5)$$

Indeed, in the representation of the basis states $|m\rangle \equiv \cos(m\theta)/\sqrt{\pi}$ and for an effective Planck constant $\hbar$, the DKR propagator takes the familiar form $\hat{v}(k)\hat{u}(\hbar)$ with

$$\langle m|\hat{u}(\hbar)|n\rangle = \exp[-i(\hbar/2)(n-\tilde{n}_0)^2]\delta_{mn} \quad (6)$$

and

$$\langle m|\hat{v}(k)|n\rangle \approx i^{(m-n)} J_{(m-n)}(k) \quad (7)$$

with ($k = K/\hbar$). Comparing these two systems, it is clear that upon the mapping

$$|\mathbf{m}\rangle \leftrightarrow |m\rangle, \quad (8)$$

$$2JT_0 \leftrightarrow k, \quad (9)$$

$$n_0 \leftrightarrow P_0/\hbar, \quad (10)$$

$$C_j \leftrightarrow \hbar, \quad (11)$$

the many-body spin chain dynamics is mapped to that of DKR [14,15,17], i.e., the motion of a spin wave packet along the spin chain is mapped to DKR quantum diffusion dynamics in its m space. Hence we can, whenever possible, shed light on the former by considering aspects of the latter, e.g., quantum resonance, Kolmogorov-Arnold-Moser (KAM) curves in phase space, etc. More significantly for this work, as shown below, it allows us to use tools from the control of quantum DKR dynamics [18–20] to manipulate states in the spin chain. Further, this mapping between spin chain and DKR allows us to go beyond the parameter regime confined by the true DKR (discussed below).

III. GENERATION OF SPIN WAVE PACKETS

In the context of the wave packet approach to quantum-state transfer, we now consider the first issue on spin wave packet generation [21]. Given a small number of basis states that could be used for encoding the state of a qubit, what initial superposition states should be exploited to induce the creation of quantum wave packets with slow dispersion? Here this interesting question is considered in the absence of an external field, where the system propagator is given by $\hat{V}(2JT_0)$. Remarkably, the associated DKR analogy now becomes a case of quantum resonance with $\hbar = 4\pi$, with a propagator analogously given by

$$\hat{v}(k = 2JT_0) = \exp[i(2JT_0)\cos(\theta)]. \quad (12)$$

Using this connection, the issue becomes to find initial superposition states within a given small subspace, such that the evolving quantum state remains well localized. At first glance this "localization" requirement seems too demanding because the main feature of quantum resonance dynamics is ballistic diffusion in the DKR m space. However, as we have discovered, this can still be obtained by maximizing a diffusion rate of DKR. Qualitatively speaking, for superposition states maximizing a quadratic diffusion rate introduced below, the ensuing dynamics will push outwards as much as possible the excitation profile in the m space, thus generating two well-separated wave packets with almost zero amplitude in between.

Quantitatively, let us first define the diffusion rate operator as

$$\hat{D} = \lim_{t\to+\infty} \frac{\hat{E}(t) - \hat{E}(0)}{t^2}, \quad (13)$$

where $\hat{E}(t)$ is the energy operator for the free rotor in the Heisenberg representation. For the quantum resonance case considered here one obtains

$$\hat{D} = A\sin^2(\theta), \quad (14)$$

where A is a constant. Note that $\hat{D}$ only couples states $|m\rangle$ of the same parity. Consider now a sample case where a superposition state

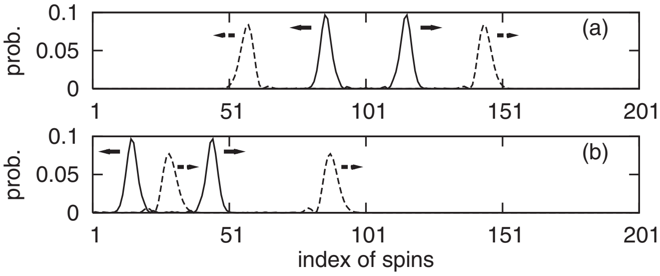

FIG. 1. Emergence of a wave packet pair in a Heisenberg chain of 201 spins, shown with the projection probability of the many-body wave function onto basis states $|\mathbf{m}\rangle$. The initial condition is a superposition state $|\psi_{m_0}\rangle$ given by Eq. (15), for (a) $m_0=101$ and (b) $m_0=30$. The system wave function then evolves, with its shape given by the solid lines at time t_1 with $2Jt_1=15$ and by the dashed lines after an additional period t_2, with (a) $2Jt_2=30$ and (b) $2Jt_2=45$. The arrows show the travel direction of the wave packets.

$$|\psi_{m_0}\rangle = \sum_{n=-2}^{n=2} \beta_{2n}|m_0 + 2n\rangle \tag{15}$$

is exploited to encode a qubit state, i.e., only five basis states are used here. The state with the largest diffusion rate, denoted D, is simply given by the eigenfunction of $\hat{D}$ in the subspace of $|m_0+2n\rangle$ $(-2\leq n\leq 2)$ with the largest eigenvalue. In particular, if these basis states do not involve state $|0\rangle$, then the maximized D is attained if $\beta_0=0.577$, $\beta_{-2}=\beta_{+2}=-0.5$, and $\beta_{-4}=\beta_{+4}=0.289$.

The significance of such an initial superposition state with maximized D is demonstrated in Fig. 1(a), with $m_0=101$ for a 201-spin chain. In particular, a well-separated wave packet pair is seen to quickly emerge, and its dispersion after its emergence is impressively slow in the absence of any external static fields. Note that, unlike the accelerator mode approach proposed in Ref. [14], the wave packet pair is created by the system dynamics itself. Note also that the excitation amplitudes between the two wave packets are surprisingly small. Because the total polarization here is fixed, a well-separated wave packet pair directly indicate quantum entanglement between well-separated parts of the spin chain, and their travel in opposite directions distributes information or entanglement to both ends of the spin chain. Further results (not shown) indicate that in the case of minimized D or an arbitrarily chosen initial state, the system dynamics generically creates a quickly delocalizing state (note also that even in chaotic cases different initial superposition states may also lead to dramatic differences in the ensuing quantum diffusion dynamics [22]). These further demonstrate the important role of an initial superposition state with a maximized diffusion rate in encoding the quantum information. Certainly, if more basis states are allowed in encoding the quantum information, then wave packet pairs with even slower dispersion can be created with the same approach.

It is also desirable to be able to create a well-separated wave packet pair that transfers information to a common end of the spin chain. For example, if two wave packets with identical shape can be created, then one of them may be analogous to a "backup" copy as the other is being transferred and received first. Note that this possibility is not in violation of the quantum no-cloning theorem, because here the two wave packets do not independently describe the quantum state of the involved spins. Rather, the two wave packets describe the entanglement between two particular sections of the spin chain.

The creation of such a wave packet pair is achieved here by going beyond the kicked rotor perspective and exploiting the boundary effect associated with the spin chain. That is, we apply the above scenario, but with the initial encoding state $|\psi_{m_0}\rangle$ located at $m_0 < N/2$, and with the requirement that no information receiver presents at the left $(n=1)$ end. To be more specific, consider a sample result shown in Fig. 1(b), with $m_0=30$. The wave packet creation dynamics in the early stage is seen to be analogous to the case of Fig. 1(a). Sometime later, the generated wave packet moving to the left hits the boundary and gets reflected. As demonstrated in Fig. 1(b), this then creates a pair of wave packets where both of the members of the pair are moving to the right, with their shape indistinguishable from one another, with almost zero excitation in-between, and a peak-to-peak distance of $2(m_0-1)$ spins.

IV. STOPPING AND RELAUNCHING WAVE PACKET PROPAGATION

In this section, taking the wave packets obtained in the previous section as examples, we shall consider an independent issue, namely, stopping and relaunching the quantum-state transfer along a spin chain. A parabolic magnetic field [see Eq. (1)] is proposed as the control field with a simple global feature, and we aim to achieve our control objective with an always-on Heisenberg interaction.

Thanks to the DKR analogy discussed above, the problem is now converted into the question of how to stop and successfully restart the transport process in the DKR m space. We do so below by exploiting control features of the DKR system. Two features are relevant, one classical and one quantal.

The classical basis of our control scenario arises by exploiting the phase space KAM curves of the underlying classical dynamics. That is, if the kicking magnetic field is sufficiently frequent that the chaoticity parameter $|2JT_0C_j|$ in the classical DKR is sufficiently small, the underlying classical dynamics will be mainly integrable and the associated KAM curves will present strong barriers to the quantum transport in the m space. Because KAM curves will be almost everywhere, these classical structures can effectively stop the travel of arbitrary and unknown quantum wave packets.

Figure 2 displays the fate of the moving wave packet pair shown in Fig. 1(a) (solid lines) after a kicking parabolic field is introduced. As is clearly seen in Figs. 2(a) and 2(b), the transfer of the wave packet pair to both ends of the spin chain is stopped. This dynamical effect can also be understood as a type of quantum Zeno effect achieved by frequently applying (but far from infinitely fast) external pulses to a system.

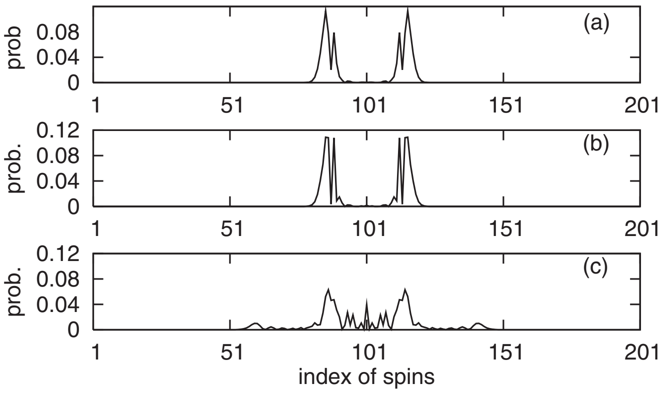

FIG. 2. Stopping of quantum-state transfer in a spin chain by a parabolic kicking field, with $C_j=0.5$. The initial state, the meaning of the wave function profile, and the spin chain used for calculations are the same as in Fig. 1(a). $2JT_0=0.25$, and the first δ kick comes at time t_1 with $2Jt_1=15$. (a) and (b) are the results after 100 and 200 kicks. The kicking field is then turned off and (c) displays the state after an additional period t_2 with $2Jt_2=30$.

However, although the wave packet pair in Figs. 2(a) and 2(b) has stopped moving, its internal structure is seen to be changing in a subtle manner. This indicates that evolution of the quantum phases characterizing the stopped wave packets is still not frozen. This fact turns out to be disastrous when the kicking field is turned off in order to relaunch the state transfer. For example, Fig. 2(c) displays the wave function after the kicking field has been off for a period of t_2 with $2Jt_2=30$: The background fluctuation is greatly increased, and the main peaks of the wave packet pair do not move further.

Hence, using KAM curves alone, which is a purely classical control mechanism, does not offer a satisfactory means of stopping the wave packet. To improve the control one must compensate for the quantum phases that are accumulated during the stopping process. This phase accumulation is due to the spin-spin interaction as well as the kicking field.

Consider then an important observation made in our previous work on the quantum control of DKR dynamics [18,20], i.e.,

$$\begin{aligned}\langle m|\hat{v}(k)|n\rangle &= \frac{1}{\pi}\int_0^{2\pi}\cos(m\theta)\exp[ik\cos(\theta)]\cos(n\theta)d\theta\\ &=\langle m|(-1)^m\hat{v}(-k)(-1)^n|n\rangle\\ &=\langle m|\hat{u}(2\pi)\hat{v}(-k)\hat{u}(2\pi)|n\rangle. \end{aligned} \tag{16}$$

The first line in Eq. (16) holds by definition, the second line becomes obvious if the integration variable θ is changed to $\theta+\pi$, and the last line is obtained by use of the definition of $\hat{u}(\hbar)$. Equation (16) hence proves

$$\hat{v}(k)=\hat{u}(2\pi)\hat{v}(-k)\hat{u}(2\pi). \tag{17}$$

Returning to a finite spin chain system, this result indicates that

TABLE I. The j dependence of C_j [see Eq. (1)] in an explicitly designed pulse sequence for the stopping of arbitrary wave packets for a period of $2M$ kicks. C is a constant discussed in the text. Note that some system parameters used here are beyond what is allowed in a true kicked rotor system.

j	1	$(1,M]$	$M+1$	$(M+1,2M]$	$2M+1$
C_j	$C/2+2\pi$	C	2π	$-C$	$-C/2$

$$\hat{V}(2JT_0)\approx\hat{U}(2\pi)\hat{V}(-2JT_0)\hat{U}(2\pi). \tag{18}$$

That is, the sign of the intrinsic interaction constant J can be effectively reversed if we apply two parabolic δ kicks of particular strength. As such, it becomes possible to compensate for the quantum phase evolution inherent in the spin chain. As to the quantum phases induced by the kicking field, they can also be compensated for by considering kicking fields with the sign of C_j reversed.

Given these considerations we present in Table I an explicitly designed special pulse sequence that can relaunch stopped wave packets with perfection. For this special pulse sequence, the KAM curves associated with small $|2JT_0C|$ still play a key role because they directly prevent the state transfer, in the same manner as demonstrated in Fig. 2. What is remarkable now is the total time evolution operator associated with the entire stopping process. In terms of the DKR analogy, this operator can be written as (after some manipulation)

$$\begin{aligned}&[\hat{u}(-C/2)\hat{v}(k)\hat{u}(-C/2)]^M[\hat{u}(C'/2)\hat{v}(k)\hat{u}(C'/2)]^M\\ &\quad=[\hat{u}(-C/2)\hat{v}(k)\hat{u}(-C/2)]^M[\hat{u}(C/2)\hat{v}(-k)\hat{u}(C/2)]^M\\ &\quad=[\hat{u}(-C/2)\hat{v}(k)\hat{u}(-C/2)]^{M'}[\hat{u}(C/2)\hat{v}(-k)\hat{u}(C/2)]^{M'}\\ &\quad=\cdots=1, \end{aligned} \tag{19}$$

where $C'=C+4\pi$, $M'=M-1$. In obtaining Eq. (19) we have used

$$\hat{u}(C)=\hat{u}(C/2+2\pi)\hat{u}(C/2+2\pi) \tag{20}$$

and Eq. (16). Equation (19) proves that at the end of the stopping time all properties characteristic of an *unknown* quantum wave packet can be exactly restored. This exact rephasing indicates that the dynamical evolution associated with the second $M/2$ kicks, in addition to offering a dynamical barrier to stop the quantum transport, precisely reverses the evolution associated with the first $M/2$ kicks. As such, the stopping is entirely *nondestructive*, as long as the system is not subject to noise effects during the stopping process. Evidently then, wave-packet-assisted information transfer can be perfectly relaunched as the kicking field is turned off. This theoretical result applies exactly to an infinitely long spin chain. But fortunately, as also demonstrated below, it applies extremely well to a finite-length chain. Note also that the designed pulse sequence in Table I is a significant extension beyond a true DKR system because both positive and negative "$\hbar$" ($\hbar\leftrightarrow C_j$) are exploited here.

In parallel with Fig. 2, Fig. 3 displays a computational example using the pulse sequence given in Table I. As in Fig.

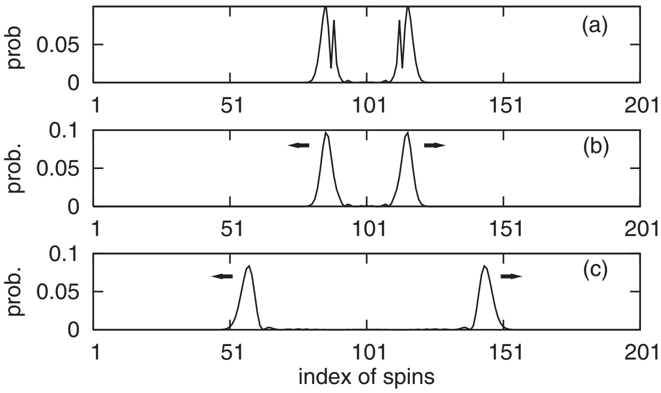

FIG. 3. Same as in Fig. 2, but for an explicitly designed pulse sequence given in Table I, with $C=0.5$ and $M=100$. The transfer of the wave packet pair along the spin chain is stopped for $2M$ kicks. Panels (a) and (b) are the results after 100 and 200 kicks. The result in (c) shows that the state transfer is indeed relaunched with perfection.

2, right before the first kick the quantum state is the wave packet pair described by the solid lines in Fig. 1(a). Figure 3(a) confirms that the wave packet pair is not moving as the special pulse sequence is on. Figure 3(b) shows that at the end of the stopping period, we have restored the initial condition [compare Fig. 3(b) with the solid lines in Fig. 1(a)]. The restoration fidelity in the numerical calculations for a 201-spin chain is found to be higher than $[1-10^{-13}]$. The kicking field is then turned off. As expected, quantum-state transfer is relaunched and the wave packets continue their journey, with slow dispersion, towards both ends of the spin chain [Fig. 3(c)]. Indeed, results in Fig. 3(c) are indistinguishable from the dashed lines in Fig. 1(a).

The control scenario proposed in this work can also lead to other very interesting approaches to the manipulation of quantum entanglement dynamics of a spin chain. Here we briefly discuss three possibilities. First, by modifying the kicking field profile we can choose to stop only one component of a wave packet pair, e.g., of the pair shown in Fig. 1(b) with dashed lines, thereby offering an interesting method of tuning the time delay between two wave packets moving in the same direction. This then offers a means of controlling the distance between two entangled parts of the spin chain. Second, because the sign of the intrinsic spin-spin interaction constant J can be effectively reversed if we apply δ kicks of particular strength, it can be easily shown that one can bounce back an arbitrary and unknown moving wave packet to the sender at a time of our choosing. Third, by controlling the time delay between the two wave packets and/or taking advantage of the feasibility of time reversal, we may also recombine two localized wave packets at a location different from that of the initial state. This recombination dynamics resembles that of a double-slit experiment, thereby generating interesting interference patterns along the spin chain. Such kind of interference patterns of spin excitations, and their fate under a variety of circumstances, may work as a novel interferometer for fundamental studies in quantum physics.

V. CONCLUSION

To conclude, based on a mapping between a kicked spin chain and the delta-kicked rotor system [14], we have shown that previous quantum control results in the delta-kicked rotor system [18–20,22] can be applied to the control of spin wave packet propagation and hence the control of propagating quantum information encoded in wave packets. Specifically, we have proposed a simple approach to wave packet creation in a Heisenberg spin chain and demonstrated the possibility of stopping and relaunching information transfer without individually addressing spins or turning off spin-spin interactions. Several interesting applications of this work in manipulating the dynamics of a spin chain are also discussed. The results indicate that many insights from the quantum chaos research can be very useful for quantum information science. This work also adds more support to the use of spin chains as quantum wires, and might be useful in designing new quantum computation algorithms with an always-on qubit-qubit interaction [2,3]. Extensions to other types of spin chains are under consideration.

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