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Asoka Biswas, Moshe Shapiro and Paul Brumer



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Overlapping resonances in the resistance of superposition states to decoherence

Asoka Biswas,¹ Moshe Shapiro,² and Paul Brumer¹

¹*Department of Chemistry, Chemical Physics Theory Group, and Center of Quantum Information and Quantum Control, University of Toronto, Toronto, Ontario M5S 3H6, Canada*

²*Department of Chemistry, University of British Columbia, Vancouver V6T 1Z1, Canada and Department of Chemical Physics, The Weizmann Institute of Science, Rehovot 76100, Israel*

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Overlapping resonances are shown to provide new insights into the extent of decoherence experienced by a system superposition state in the regime of strong system-environment coupling. As an example of this general approach, a generic system comprising spin-half particles interacting with a thermalized oscillator environment is considered. We find that (a) among the collection of parametrized Hamiltonians, the larger the overlapping resonances contribution, the greater the maximum possible purity, and (b) for a fixed Hamiltonian, the larger the overlapping resonances contribution, the larger the range of possible values of the purity as one varies the phases in the system superposition states. Systems displaying decoherence free subspaces show the largest overlapping resonances contribution. © 2010 American Institute of Physics.

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I. INTRODUCTION

Coherent control of atomic and molecular processes is a subject of considerable ongoing interest.^{1,2} Specifically, a number of successful control scenarios have been theoretically proposed and experimentally implemented. The vast majority of well characterized scenarios are, however, based on molecules treated in isolation, with the understanding that systems in an external environment are subject to decoherence effects^{3,4} that can be deleterious to control.⁵ Decoherence effects of this kind are also detrimental to “quantum technologies,” such as quantum information processing and quantum computing⁶ insofar as they cause the loss of the essential quantum characteristics that make these research areas interesting.

As a consequence, efforts are ongoing to find suitable mechanisms to reduce or compensate for decoherence effects. For example, several techniques, such as quantum dynamical decoupling⁷ (QDD) and the use of decoherence free subspace (DFS),⁸ have been proposed in the quantum information community. However, the QDD method is quite challenging to implement using the present technology, and the DFS requires explicit symmetries in the Hamiltonian, which are not generic. Hence, methods for identifying conditions for states that are resistant to decoherence are of great interest.

Indeed, considerable effort has gone into identifying characteristic features of particular system states, interacting with different environments that lead to stability against decoherence. Sample approaches include the general predictability sieve approach of Zurek *et al.*⁹ and predictions of the unusual stability of coherent matter states, low lying eigenstates, and localized states^{10,11} for typical environmental coupling.

In this paper we introduce a new perspective, based on

overlapping resonances, to the understanding and optimization of the resistance of system superposition states to decoherence. The approach applies in the important regime where system-environment coupling is large. Specifically, we show that the interference between the overlapping resonances that exist in this regime can be used to build system superposition states that are relatively resistant to decoherence. The extent to which such states are stable against decoherence is shown to directly correlate with the degree to which the system-environment Hamiltonian displays overlapping resonances. Thus, even if the system does not possess symmetry properties such as DFSs, states can be constructed which are more resistant to decoherence than are others.

From a formal viewpoint, the approach described herein constitutes a major extension, to product Hilbert spaces characteristic of open quantum systems, of our work on controlling radiationless transitions in large molecules.¹² There, population transfer between a Q -subspace and the remainder of the Hilbert space, the P -subspace, was minimized by preparing initial superpositions in the Q -subspace, relying on the presence of overlapping resonances. This approach was applicable to the case where the full Hilbert space was partitioned into the *sum* of two subspaces. By contrast, decoherence involves the effect of an environment on the system, i.e., a *product* of two Hilbert spaces. The difference is substantial and significant.

The inter-relationship between overlapping resonances and the ability to design states that are resistant to decoherence is general. It proves useful, however, to introduce the approach via a specific generic example. For this reason we choose a particular class of spin-boson problems. Note that system decoherence is a manifestation of the loss of quantum information when one neglects the environment with which

the system is entangled.⁴ As such, decoherence can occur effectively even with a small environment¹³ as is manifested in the example below.

The structure of the paper is as follows. In Sec. II, we introduce a generic system that describes a spin system interacting with an environment, providing a specific example for this general approach. Section III considers the nature of decoherence in this system, and its relationship to overlapping resonance is explored in Sec. IV. In particular, Sec. IV B provides a qualitative argument for the role of overlapping resonances in decoherence control. Some specific numerical examples are provided in subsections of Sec. IV, demonstrating correlations between overlapping resonances and reduced state decoherence. Section V provides a summary.

II. A GENERIC CLASS OF HAMILTONIANS

The spin-boson problem has been extensively studied in Chemistry in various scenarios and for a wide variety of applications.¹⁴ Here we consider, as a specific example, the control of decoherence in a simple spin system interacting with a small thermal environment, i.e., a single spin-half system and an environment comprising a single-mode oscillator. The most general system-environment Hamiltonian for this case is given by

$$H/\hbar = \omega a^\dagger a + \frac{\omega_0}{2} S_z + g_r (S_+ a + S_- a^\dagger) + g_{nr} (S_+ a^\dagger + S_- a) + g_{ph} S_z (a + a^\dagger), \quad (1)$$

where a and $a^\dagger$ are the environment annihilation and creation operators, ω is the frequency of the environment mode, $\hbar\omega_0$ is the energy difference between the two spin-half states $|\pm\rangle$, and the spin operators are defined as $S_\pm = |\pm\rangle\langle\mp|$, $S_z = |+\rangle\langle+| - |- \rangle\langle-|$. Note that the rotating wave approximation (RWA) has not been invoked: The g_r term in the Hamiltonian is the spin-environment interaction that is retained under the RWA, whereas the g_{nr} term is that which would be neglected in the RWA. The g_{ph} term corresponds to decoherence of the spin due to the operator S_z , which induces a relative phase between the spin states $|+\rangle$ and $|- \rangle$. The g_r and g_{nr} terms mediate energy exchange between the spin and the environment. Our focus below is in the region of strong coupling, where g_{nr} and g_r are both greater than ω_0 . It is important to emphasize that although the environment is small (a single oscillator), its coupling to the system does induce significant system decoherence, as manifested in computations below.

The class of Hamiltonians [Eq. (1)] possesses a plane defined by $g_r = g_{nr}$ in the three dimensional g_{nr}, g_r, g_{ph} parameter space upon which the system displays a DFS. That is, in these special cases the Hamiltonian Eq. (1) leads to a DFS due to a Lie algebraic symmetry. In these cases, the interaction part of the total Hamiltonian can be written as $\mathcal{E}_I \otimes B_I$, where $B_I = (a + a^\dagger)$ is the environment operator, and the spin operator $\mathcal{E}_I$ can be written in a closed matrix form in the $(|+\rangle, |- \rangle)$ spin basis,

$$\mathcal{E}_I = \begin{pmatrix} g_{ph} & g \\ g & -g_{ph} \end{pmatrix}, \quad g = g_r = g_{nr}. \quad (2)$$

The DFS corresponds to the eigenstates of the $\mathcal{E}_I$ matrix. For other values of g_{nr} and g_r , of primary interest here, the system does not have a DFS.

III. DECOHERENCE OF A SPIN SUPERPOSITION

We consider the stability against decoherence of an initial spin superposition state $|\psi\rangle = c_+|+\rangle + c_-|-\rangle$, where $c_\pm$ are complex amplitudes and the environment is in thermal equilibrium at temperature T . The latter is described in terms of the oscillator eigenstates $|n\rangle$ by the environment density matrix $\rho_B = \sum_n p_n |n\rangle\langle n|$, $p_n = e^{-E_n/k_B T} / (\sum_n e^{-E_n/k_B T})$, and $E_n = (n + 1/2)\hbar\omega$. In order to follow the temporal evolution of the system we expand the total (system+environment) evolution operator U in terms of $|\gamma\rangle$, the eigenstates of the full Hamiltonian [i.e., $(E_\gamma - H)|\gamma\rangle = 0$] with $U = \sum_\gamma e^{-iE_\gamma t/\hbar} |\gamma\rangle\langle\gamma|$. Expanding the eigenstates $|\gamma\rangle$ in the zeroth order basis of “bare” spin+environment states $\{|\pm, n\rangle \equiv |\pm\rangle \otimes |n\rangle\}$, (i.e., the eigenstates of H without the spin-environment interaction) gives, for the time-evolved density matrix of the total spin+environment system,

$$\begin{aligned} \rho(t) &= \sum_n p_n U[|\psi\rangle\langle\psi| \otimes |n\rangle\langle n|] U^\dagger \\ &= \sum_{\gamma, \gamma', n} p_n e^{-i(E_\gamma - E_{\gamma'})t} |\gamma\rangle\langle\gamma'| \langle\gamma|\psi, n\rangle\langle\psi, n|\gamma'\rangle. \end{aligned} \quad (3)$$

Further, tracing over the environment states, gives the following reduced system density matrix elements $(\rho_s)_{kl}$ in the basis of spin states $|k\rangle$ and $|l\rangle$, where k, l denote either the plus or minus state,

$$\begin{aligned} (\rho_s)_{kl} &= \sum_{m, n, \gamma, \gamma'} p_n e^{-i(E_\gamma - E_{\gamma'})t} \langle k, m|\gamma\rangle\langle\gamma|\psi, n\rangle\langle\psi, n|\gamma'\rangle \\ &\quad \times \langle\gamma'|l, m\rangle. \end{aligned} \quad (4)$$

Expanding $|\psi\rangle$ as $c_+|+\rangle + c_-|-\rangle$, we can rewrite the above expression as

$$(\rho_s)_{kl} = |c_+|^2 Q_{kl}(t) + |c_-|^2 R_{kl}(t) + c_+ c_-^* P_{kl}(t) + c_- c_+^* T_{kl}(t), \quad (5)$$

where P, Q, R , and T are complex matrices with elements

$$\begin{aligned} P_{kl}(t) &= \sum_{m, n, \gamma, \gamma'} p_n e^{-i(E_\gamma - E_{\gamma'})t} \langle k, m|\gamma\rangle\langle\gamma'|l, m\rangle\langle\gamma|+\rangle\langle+|n\rangle \\ &\quad \times \langle-|n\rangle\langle\gamma'\rangle, \end{aligned}$$

$$\begin{aligned} Q_{kl}(t) &= \sum_{m, n, \gamma, \gamma'} p_n e^{-i(E_\gamma - E_{\gamma'})t} \langle k, m|\gamma\rangle\langle\gamma'|l, m\rangle\langle\gamma|+\rangle\langle+|n\rangle \\ &\quad \times \langle+|n\rangle\langle\gamma'\rangle, \end{aligned}$$

$$\begin{aligned} R_{kl}(t) &= \sum_{m, n, \gamma, \gamma'} p_n e^{-i(E_\gamma - E_{\gamma'})t} \langle k, m|\gamma\rangle\langle\gamma'|l, m\rangle\langle\gamma|-\rangle\langle-|n\rangle \\ &\quad \times \langle-|n\rangle\langle\gamma'\rangle, \end{aligned}$$

$$T_{kl}(t) = \sum_{m,n,\gamma,\gamma'} p_n e^{-i(E_\gamma - E_{\gamma'})t} \langle k, m | \gamma \rangle \langle \gamma' | l, m \rangle \langle \gamma | -, n \rangle \times \langle +, n | \gamma' \rangle. \quad (6)$$

Consider the decoherence of the spin state, quantified in terms of the widely adopted standard of purity^{4,15} S of the spin, where $S = \text{Tr}(\rho_s^2)$. Given the above results, S can be evaluated via

$$S = \sum_{k,l \in +,-} \langle k | \rho_s | l \rangle \langle l | \rho_s | k \rangle = \sum_{k,l \in +,-} |(\rho_s)_{kl}|^2 \quad (7)$$

so that using $P_{kl}^* = T_{lk}$,

$$\begin{aligned} S = & |c_+|^4 \sum_{k,l \in +,-} |Q_{kl}|^2 + |c_-|^4 \sum_{k,l \in +,-} |R_{kl}|^2 \\ & + |c_+|^2 |c_-|^2 \sum_{k,l \in +,-} [2|P_{kl}|^2 + \{Q_{kl}R_{kl}^* + Q_{kl}^*R_{kl}\}] \\ & + 2 \text{Re} \left[\sum_{k,l \in +,-} [c_+^2 c_-^* P_{kl} P_{lk} + |c_+|^2 c_+ c_- [Q_{kl} P_{kl}^* \right. \\ & \left. + P_{lk}^* Q_{kl}^*] + |c_-|^2 c_+ c_- [R_{kl} P_{kl}^* + P_{lk}^* R_{kl}^*]] \right]. \end{aligned} \quad (8)$$

The first three of these terms depend solely on the magnitude of the $c_\pm$ coefficients, whereas the remainder of the terms depends on the phases of the coefficients as well. The latter are clear contributions to S that rely upon quantum interference between the spin states.

Note, for comparison later below, that $S=1$ denotes a pure state. If the state is completely decohered in the spin basis then $(\rho_s)_{kl} = \frac{1}{2} \delta_{kl}$ so that the purity, by Eq. (7), is $S = 2(1/2)^2 = 1/2$.

With the P , Q , R , and T matrices known, one can optimize S with respect to c_+ and c_- to determine the state with maximum purity S at a fixed time. In particular, the relative phase of c_+ and c_- is important if either Q_{kl} or R_{kl} is nonzero, and P_{kl} is nonzero. Although not evident from Eq. (8), numerical results below clearly demonstrate a reliance on this relative phase to maximize S . Further, the nature of Q_{kl} , R_{kl} , and P_{kl} is intimately related to overlapping resonances, as discussed below.

IV. OVERLAPPING RESONANCES AND SYSTEM PURITY

A. Overlapping resonances

As noted above, the bare states $|\pm, n\rangle$ are not eigenstates of H , and will evolve in the presence of the system-environment coupling. Insight into the nature of this time evolution is afforded by expanding the zeroth order state in the exact eigenstates $|\gamma\rangle$. For example, a bare state such as $|+, j\rangle$ expands as

$$|+, j\rangle = \sum_{\gamma} |\gamma\rangle \langle \gamma | +, j \rangle, \quad (9)$$

and evolves as

$$|+, j\rangle_t = \sum_{\gamma} |\gamma\rangle \langle \gamma | +, j \rangle e^{-iE_{\gamma}t/\hbar}. \quad (10)$$

The E_{γ} dependence of the square of the expansion coefficient, i.e., $|D_{\gamma}|^2 \equiv \langle \gamma | +, j \rangle^2$ provides the energy width over which the zeroth order state $|+, j\rangle$ is spread due to the system-environment interaction. In the simplest cases the inverse of this width provides a qualitative measure of the time scale for the evolution of $|+, j\rangle$. Hence, the zero order states are indeed resonances with a characteristic width $|D_{\gamma}|^2$. This is the analog, in a bound state spectrum, of the well known resonance in the continuum.

Similarly, by analogy to the continuum case, we can define *overlapping resonances*, as resonances that overlap in energy space, i.e., resonances that share a common $|\gamma\rangle$ in their respective eigenstate expansions. These are the precise bound state analogs to overlapping resonances in the continuum (e.g., Fig. 1, Ref. 16). Examples of bound state overlapping resonances in molecular systems have also been previously reported.¹²

In the case of the spin-boson Hamiltonians given by Eq. (1), overlapping resonances were clearly evident in, but not limited to, the strong coupling regime characterized by g_r and g_{nr} greater than ω_0 . They occur as two states $|k, m\rangle$ and $|l, n\rangle$ with $k \neq l$ and/or $m \neq n$ ($k, l \in +, -$) that have nonzero overlap with the same eigenstate $|\gamma\rangle$. Examples of such overlapping resonances are shown in Fig. 1. Quite clearly the overlap between the resonances can be significant, enhanced by the fact that the overall width of each resonance can be significantly larger than the energy spacing between the zeroth order states. The resonances are also seen to be rather highly structured.

B. Overlapping resonances and decoherence control: Qualitative motivation

The rather complex contribution of the overlapping resonances to the system purity is discussed in Sec. IV C below. However, qualitative motivation for the role of overlapping resonances in controlling decoherence is readily provided by considering the survival probability of an initial separable (spin+bath) state, as described below.

Consider one wave function contribution to $\rho(t)$ [Eq. (3)] which, at time $t=0$, is given by

$$|\Psi_0\rangle = |\psi\rangle |n\rangle. \quad (11)$$

Subsequent evolution of the state yields $|\Psi(t)\rangle$. Preventing decoherence, from this viewpoint, means preserving the $|\psi\rangle$ component of $|\Psi_0\rangle$ during the course of the dynamics.

Consider first then the probability $I(t) = |\langle \Psi_0 | \Psi(t) \rangle|^2$ of finding the state $|\Psi_0\rangle$ at time t . Expanding $|\Psi(t)\rangle$ in terms of $|\gamma\rangle$ would give

$$|\Psi(t)\rangle = \sum_{\gamma} |\gamma\rangle \langle \gamma | \Psi_0 \rangle e^{-iE_{\gamma}t/\hbar}. \quad (12)$$

$I(t)$ is then given by

$$I(t) = \sum_{\gamma, \gamma'} |\langle \Psi_0 | \gamma \rangle|^2 |\langle \Psi_0 | \gamma' \rangle|^2 e^{i(E_{\gamma} - E_{\gamma'})t/\hbar}. \quad (13)$$

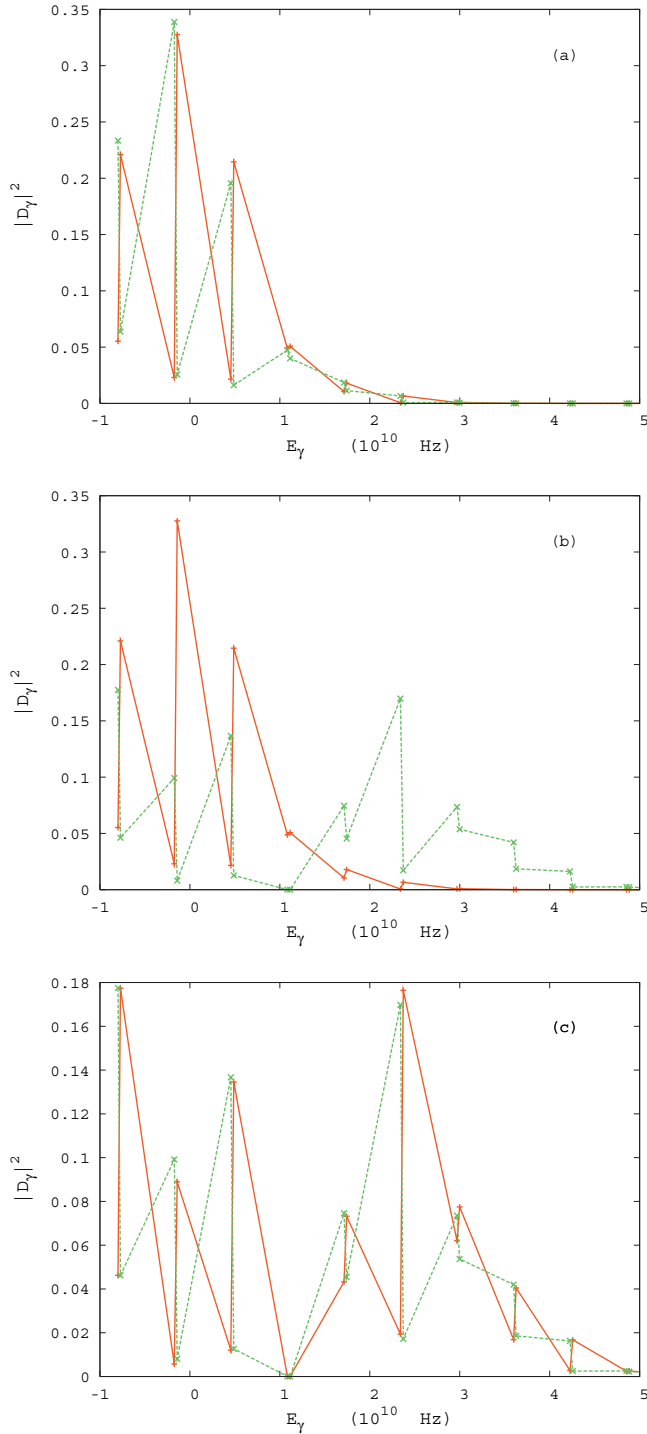


FIG. 1. Plot of the overlap $|D_\gamma|^2$ between various bare states and the eigenstates $|\gamma\rangle$ as a function of the energy E_γ : (a) $|+,0\rangle$, $|-,0\rangle$, (b) $|+,0\rangle$, $|-,2\rangle$, and (c) $|+,2\rangle$, $|-,2\rangle$. The data points for the first of each pair are connected by a solid line whereas the data points for the second of each pair are connected by solid lines. Data are for the parameters $\omega=2\pi$ GHz, $\omega_0=2\pi\times 100$ MHz, $g_r=g_{nr}=2\pi$ GHz, and $g_{ph}=0$.

To gain insight, we consider the simplified case where the time is long and the density of the energy spectrum $\{E_\gamma\}$ is large. In that case only the $\gamma=\gamma'$ term contributes so that $I(t)$ becomes

$$I(t) = \sum_{\gamma} |\langle \gamma | \Psi_0 \rangle|^4. \quad (14)$$

Consider then, for the decoherence case, an initial product state comprised of $|\Psi_0\rangle = [c_+|+\rangle + c_-|-\rangle]|n\rangle$. In this case the survival probability is given by

$$I(t) = \sum_{\gamma} |\langle \gamma | [c_+|+\rangle + c_-|-\rangle]|n\rangle|^4 \\ = \sum_{\gamma} |c_+ \langle \gamma | +, n \rangle + c_- \langle \gamma | -, n \rangle|^4. \quad (15)$$

Clearly, both terms inside the absolute value only contribute if both $\langle \gamma | +, n \rangle$ and $\langle \gamma | -, n \rangle$ are nonzero, i.e., if the resonances $|+, n\rangle$ and $|-, n\rangle$ overlap. In the event that they do not overlap, control over the survival probability is limited to two parameters, $|c_+|$ and $|c_-|$. In the presence of overlapping resonances, however, a third parameter, the relative phase of c_+ and c_- is important, guaranteeing improved control over the survival probability. Specifically, increased $I(t)$ would imply less decoherence, and vice versa.

Finally, note further that the structure of Eq. (15) is clearly that of interfering quantum contributions to the amplitude associated with the γ contribution to $I(t)$. Given this qualitative motivation, exact contributions to the standard purity measure of decoherence are discussed below in terms of their relation to overlapping resonances.

C. Contributions to $\text{Tr } \rho_s^2$

Consider then the role of these overlapping resonances in the dynamical evolution of $\rho_s(t)$ given in Eq. (5). Specifically, note that all the matrices P , Q , R , and T depend on overlapping resonance contributions. Their dependences differ, however, in the important case where $k \neq l$, $k, l \in +, -$. In that case overlapping resonances contribute to T_{kl} and P_{kl} , but not to Q_{kl} and R_{kl} . That is, the terms in $(\rho_s)_{kl}$ that are sensitive to the relative phase of c_+ and c_- [see Eq. (5)], and are hence associated with phase control over the dynamics, are directly dependent on overlapping resonances contributions. Further, from Eq. (8), it is apparent that the purity is strongly affected by overlapping resonances through the matrix elements Q_{kl} , R_{kl} , P_{kl} , and T_{kl} . Here, as above with the survival probability, control over decoherence is enhanced by the participation of the relative phase of c_+ and c_- as a control parameter.

To quantify this dependence on overlapping resonances we seek a generic expression that identifies overlapping resonances contributions to the purity that are associated with the plus and minus spin states. One qualitatively motivated expression that has proven enlightening is given by

$$A_{+-} = \sum_{m \neq n} A_+^{m,n} A_-^{m,n}, \quad A_{\pm}^{m,n} = \left[\sum_{\gamma} |\langle \pm, m | \gamma \rangle \langle \gamma | \pm, n \rangle| \right]. \quad (16)$$

Here, for a pair of oscillator states $|m\rangle$ and $|n\rangle$, $A_+^{m,n}$ is a measure of the overlap (summed over all eigenstates $|\gamma\rangle$) between the bare states $|+, m\rangle$ and $|+, n\rangle$, when the spin state is $|+\rangle$. If, for the same pair of $|m\rangle$ and $|n\rangle$, there exists non-zero overlap $A_-^{m,n}$ between the bare states $|-, m\rangle$ and $|-, n\rangle$, then A_{+-} provides a measure of the overlap between states $|+\rangle$ and $|-\rangle$ that arises through the overlap between states

$|\pm, m\rangle$ and $|\pm, n\rangle$. Note that Eq. (16) is somewhat similar to the off diagonal T_{+-} and P_{+-} terms at $t=0$.

Note that the number and character of the overlapping resonances are functions of the parameters in the total Hamiltonian, e.g., here $g_r, g_{nr}, g_{ph}, \omega$, and ω_0 . For example, in the limiting case where there is no system-environment coupling (i.e., $g_r = g_{nr} = g_{ph} = 0$), the total Hamiltonian is diagonal in the spin-boson bare state basis. The overlap between the spin-boson bare states then vanishes, and in turn, so does the overlap between the states $|\pm\rangle$. For this reason, A_{+-} is designed to be zero for the limiting case where $g_r = g_{nr} = g_{ph} = 0$, a result achieved by leaving out $n=m$ terms in Eq. (16).

Equation (16) is designed for a spin 1/2 system. For a system with N states labeled by s we can introduce the quantities

$$A_{ss'} = \sum_{m \neq n} A_s^{m,n} A_{s'}^{m,n}, \quad A_s^{m,n} = \left[\sum_{\gamma} |\langle s, m | \gamma \rangle \langle \gamma | s, n \rangle| \right], \quad (17)$$

which measure overlapping resonances between system states s and s' . A collective for all system states would then involve a sum over all s and s' . Further studies on $N > 2$ level systems are planned to fully develop this measure.

D. Computational results

Consider then the relationship between overlapping resonances as reflected in A_{+-} and decoherence as embodied in the system purity S . As an example, we examine dynamics where the environment temperature $T=25$ mK and choose to vary g_r , keeping the other Hamiltonian parameters fixed.

Numerical results are obtained by diagonalizing the Hamiltonian matrix in the zeroth order basis of bare states $|\pm, n\rangle$. At the temperature of $T=25$ mK, a total of 20 oscillator states contribute, allowing us to diagonalize a 40×40 dimensional Hamiltonian matrix to obtain the desired $E_{\gamma}, |\gamma\rangle$.

To assess the decoherence we find, for each value of g_r , the maximum possible purity $S_{\max}$ in the system at a fixed time t by optimizing the complex coefficients $c_{\pm}$. Results are shown in Fig. 2 where both $S_{\max}$ and the overlapping resonance measure A_{+-} are plotted as a function of g_r at $t=0.1$ ns. The correspondence between the behaviors of these two functions is evident, with larger A_{+-} correlating well with larger $S_{\max}$. That is, the greater the overlapping resonances contribution, the more resistant is the optimal superposition to decoherence. Note significantly that the optimally resistant state need not be, and is often not, an eigenstate of the spin Hamiltonian, but is rather a superposition of $|+\rangle$ and $|-\rangle$ eigenstates.

Prominent in the graph (and some presented below) is the DFS point $g_r = g_{nr}$, where the A_{+-} curve displays a sharp peak, commensurate with $S_{\max}$ reaching its maximum value of unity. That is, maximal purity clearly correlates with the maximum A_{+-} . The deviation of $S_{\max}$ from unity as one moves away from this point is significant, particularly in scenarios such as quantum computation, where extremely high degrees of coherence are necessary.

The results clearly show that states take advantage of overlapping resonances (and hence of quantum interference)

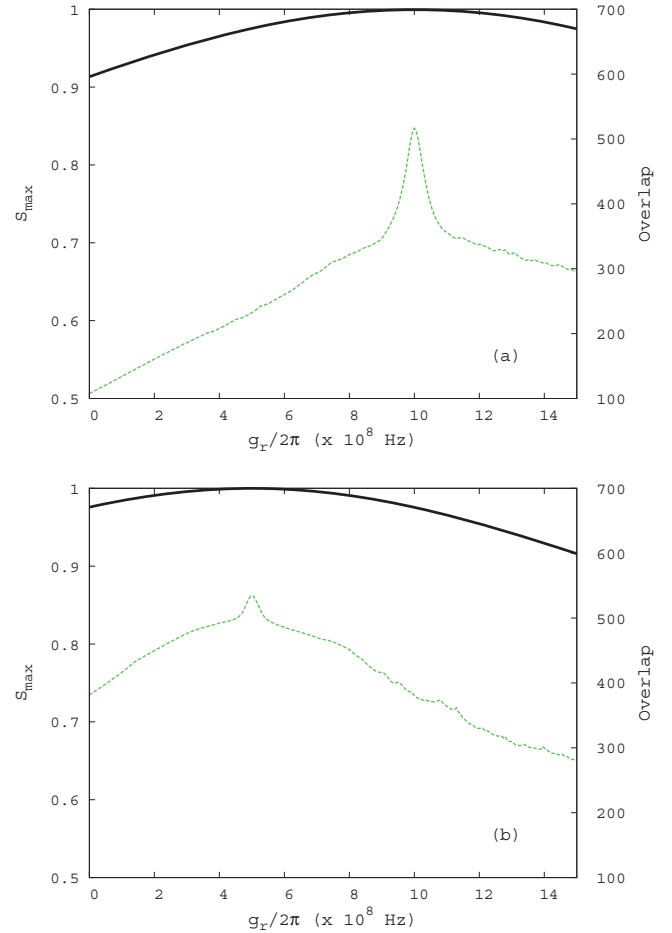


FIG. 2. The dependence of the maximum achievable purity $S_{\max}$ of the system (solid line) and the overlapping resonance A_{+-} (dashed line; right y-axis) on the coupling strength $g_r/(2\pi)$. Here (a) $g_{ph}=2\pi \times 500$ MHz, $g_{nr}=2\pi$ GHz and (b) $g_{ph}=2\pi$ GHz, $g_{nr}=2\pi \times 500$ MHz. The remaining parameters are $\omega=2\pi$ GHz, $\omega_0=2\pi \times 100$ MHz, and the temperature $T=25$ mK. The purity is calculated at a time $t=0.1$ ns.

to increase the state purity. This is the case even if the system-environment coupling is very large, as is clear from Fig. 2(b), where, for example, $g_r \sim 2\pi \times 1.5$ GHz ($g_r \gg \omega_0$), but where the purity remains as large as 0.92 for $g_{ph}=2\pi$ GHz.

The presence of overlapping resonances, as seen above, influences the degree of decoherence when preparing initial states with different initial coefficients $c_{\pm}$. Sample dependences of S on $c_{\pm}$ are shown in Fig. 3 for the case of $g_r=2\pi \times 400$ MHz, $g_{nr}=2\pi$ GHz, and $g_{ph}=2\pi \times 500$ MHz [i.e., parameters associated with Fig. 2(a)]. Shown are the time evolution of S for three states, one being the best case superposition $c_+ = -0.539998$ and $c_- = \sqrt{1 - |c_+|^2}$, which shows $S_{\max}=0.96$ at the time $t=0.1$ ns, where S is optimized. Comparison is made with the time evolution of S for two other sample states with different $c_{\pm}$ values: $c_+=1$ and $c_+=c_- = 1/\sqrt{2}$. The range of S as a function of initial state is large, with the $c_+=1$ case showing almost fully mixed behavior at long time. Interestingly, the curves shown do not cross as a function of time. Hence the optimal superposition, determined by coefficients obtained for the $t=0.1$ ns case is, in fact, the optimal superposition for the

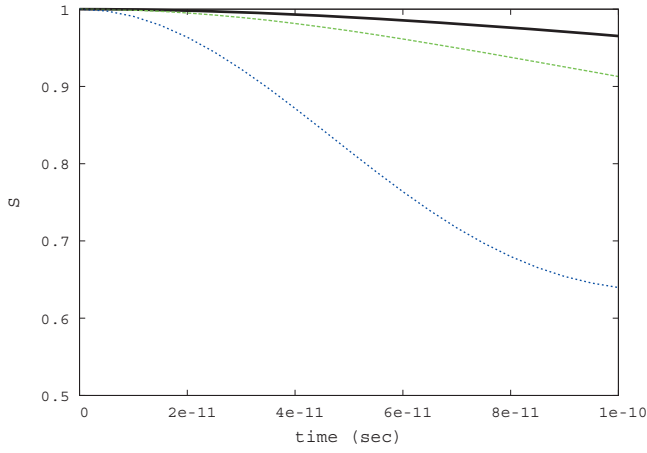


FIG. 3. Variation of purity with time for the initial states $(-0.539\,998|+\rangle + 0.841\,665|-\rangle)/\sqrt{2}$ (solid line), $(|+\rangle + |-\rangle)/\sqrt{2}$ (dashed line), and $|+\rangle$ (dotted line). Here the interaction strengths $g_r = 2\pi \times 400$ MHz, $g_{nr} = 2\pi$ GHz, and $g_{ph} = 2\pi \times 500$ MHz. The other parameters are as in Fig. 2.

times preceding that target time as well. This was also confirmed by optimizations independently carried out at shorter times.

Additional results showing how overlapping resonances between spin states are manifested in decoherence are shown in Fig. 4. Here, for a given value of g_r , we obtain the $|c_+\rangle$ and $|c_-\rangle$ that maximize S and then vary the relative phase θ between c_+ and c_- to obtain S_{diff} , defined as the difference between the maximum and minimum purity so attained. This difference is plotted in Fig. 4 as a function of g_r , along with the corresponding value of the overlapping resonance contribution A_{+-} . The extent to which S_{diff} varies is seen to reflect the variation of A_{+-} . Specifically, as A_{+-} increases, so does the extent to which the purity varies with θ . Hence, in the presence of large overlapping resonance, one can *actively* control the purity of the system by taking advantage of the phase dependent quantum interference contribution to S . Indeed, in accord with Eq. (8), an observed variation of the decoherence with the phase of a superposition state provides evidence of the presence of overlapping resonances.

The extent to which S_{diff} varies with A_{+-} depends on the

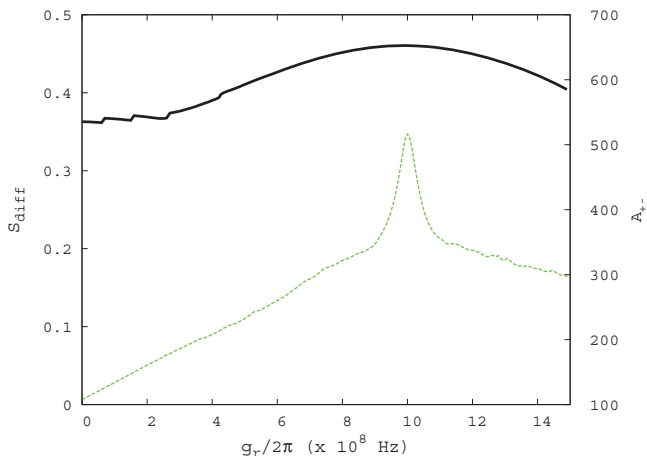


FIG. 4. Variation of S_{diff} (solid line) and the overlap A_{+-} (dashed line; right y-axis) with g_r for $g_{nr} = 2\pi$ GHz and $g_{ph} = 2\pi \times 500$ MHz. The remaining parameters are as in Fig. 2.

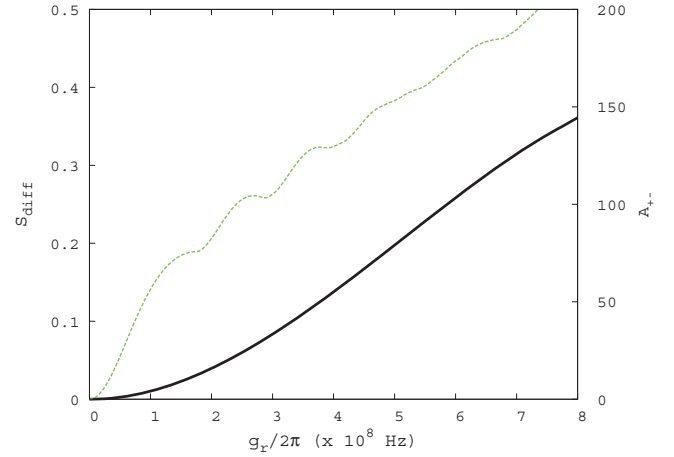


FIG. 5. Variation of S_{diff} (solid line) and the overlap A_{+-} (dashed line; right y-axis) with $g_r = g_{nr}$ for $g_{ph} = 0$. The other parameters are as in Fig. 2.

Hamiltonian parameters. Consider, for example, an interesting case of the nanoscale Hamiltonian system in Eq. (1), the interaction of a Cooper pair box (CPB) with a nanomechanical resonator.¹⁷ This system is given by

$$H_{\text{CPB}} = 4E_C \delta n S_z - (E_J/2) S_x + \hbar \omega a^\dagger a + \hbar g S_z (a + a^\dagger), \quad (18)$$

where E_C and E_J are the charging energy and the Josephson energy of the CPB, ω is the fundamental frequency of the resonator, and δn lies between $-1/2$ and $1/2$. At the degeneracy point $\delta n = 0$ this Hamiltonian can be written in the form of that of Eq. (1) under a similarity transformation, where we identify $\omega_0 = E_J$, $g_{ph} = 0$, and $g_r = g_{nr} = g$. The similarity transformation is given by the operator e^{iS_y} , with S_y being the y -projection of the spin operator.

From the symmetry of Hamiltonian (18), one finds a DFS corresponding to the state $c_+ = c_- = 1/\sqrt{2}$ and $c_+ = -c_- = 1/\sqrt{2}$. Hence, the value of S_{max} will always be unity for all values of g , irrespective of the values of A_{+-} . Nonetheless, overlapping resonances affect the decoherence in this system. Specifically, as A_{+-} increases with g , the variation of S with changes in the relative phase θ between $c_\pm$ also increases. This is shown in Fig. 5 where the difference S_{diff} between the maximum and minimum values of S and the values of A_{+-} with g are seen to be strongly correlated.

On the other hand, if $g_r \neq g_{nr}$, there is no DFS in the system. In that case, the results (not shown) behave similarly to, e.g., Fig. 2. That is, a plot of S_{max} and $A_\pm$ as a function of g_r , with $g_r \neq g_{nr}$ and $g_{ph} = 0$, shows a clear correlation between the maximum achievable value of S and the overlapping resonance contribution.

V. SUMMARY

We have introduced a new approach to considering the resistance to decoherence of system superposition states. Specifically, using the generic class of spin-boson systems as an example we have shown that overlapping resonances of the system bare states, and the associated interference between these states, provide insight into the dependence of the decoherence of system superposition states in the important

regime of strong system-environment coupling. For example, within a given class of Hamiltonians, increasing overlapping resonance contributions allow for states of higher purity. The approach is completely general, although it has been applied here to a spin 1/2 particle interacting with a single thermal oscillator. In the first instance we anticipate that increasing the environment size will increase the participation and effectiveness of overlapping resonances in decoherence control.

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